

Boundary Element Method formulation of Normal and Tangential Contact with Coulomb Friction

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Introduction

Question: Surfaces are rough. Only some asperities come into contact. We want to assess how asperity interactions alter the debris formation process during adhesive wear [1]. This poster shows our initial steps for validating a model and numerical method for adhesive frictional contact of rough surfaces.

Approach: BEM is much more efficient than FEM to solve contact between rough surfaces because only the surfaces have to be discretized. A BEM formulation with Coulomb friction has been derived and implemented.

Model

Johnson's assumption

Contact between a **rigid rough** surface and an **elastic flat** surface is considered. It is equivalent to the contact between two elastic rough surfaces with different properties under some assumptions [2].

Effective Young modulus:

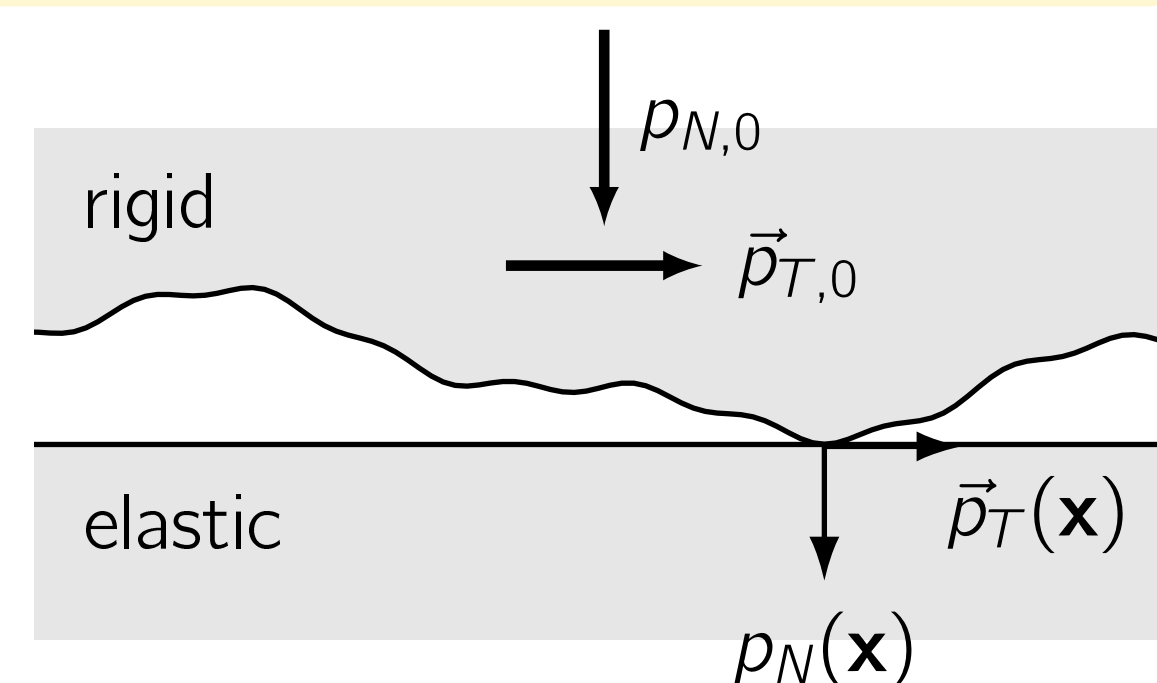
$$\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

Formulation

- Minimization of potential energy
- Coulomb friction law: $\|\vec{p}_T(\mathbf{x})\| \leq \mu p_N(\mathbf{x})$

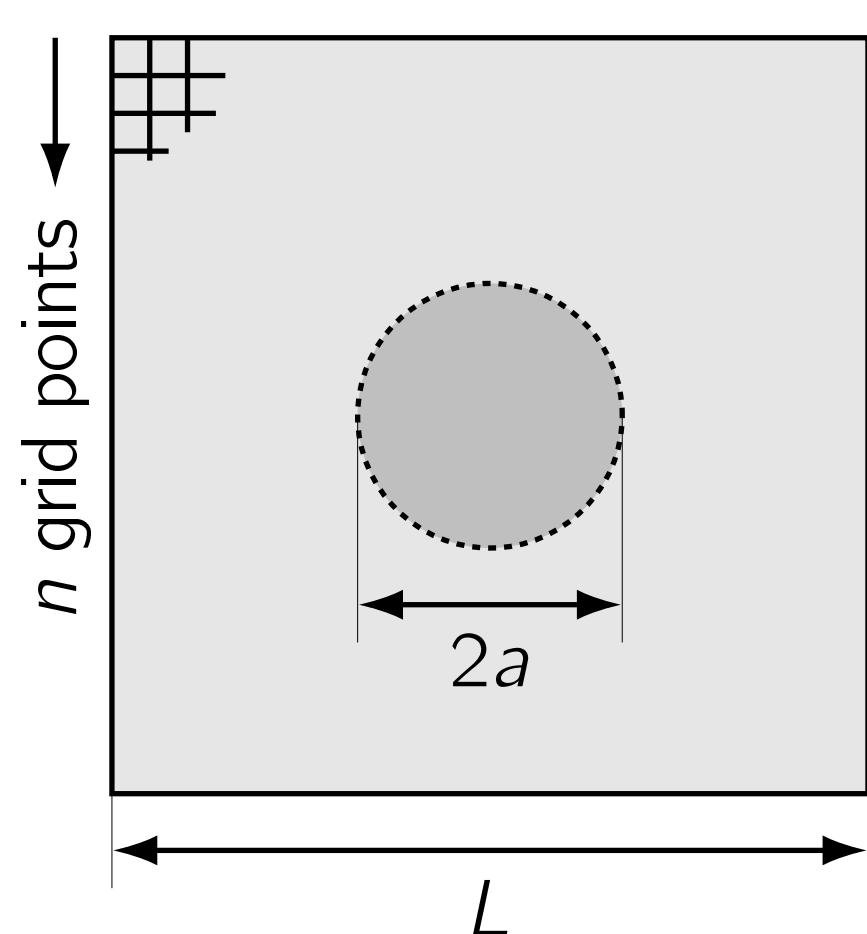
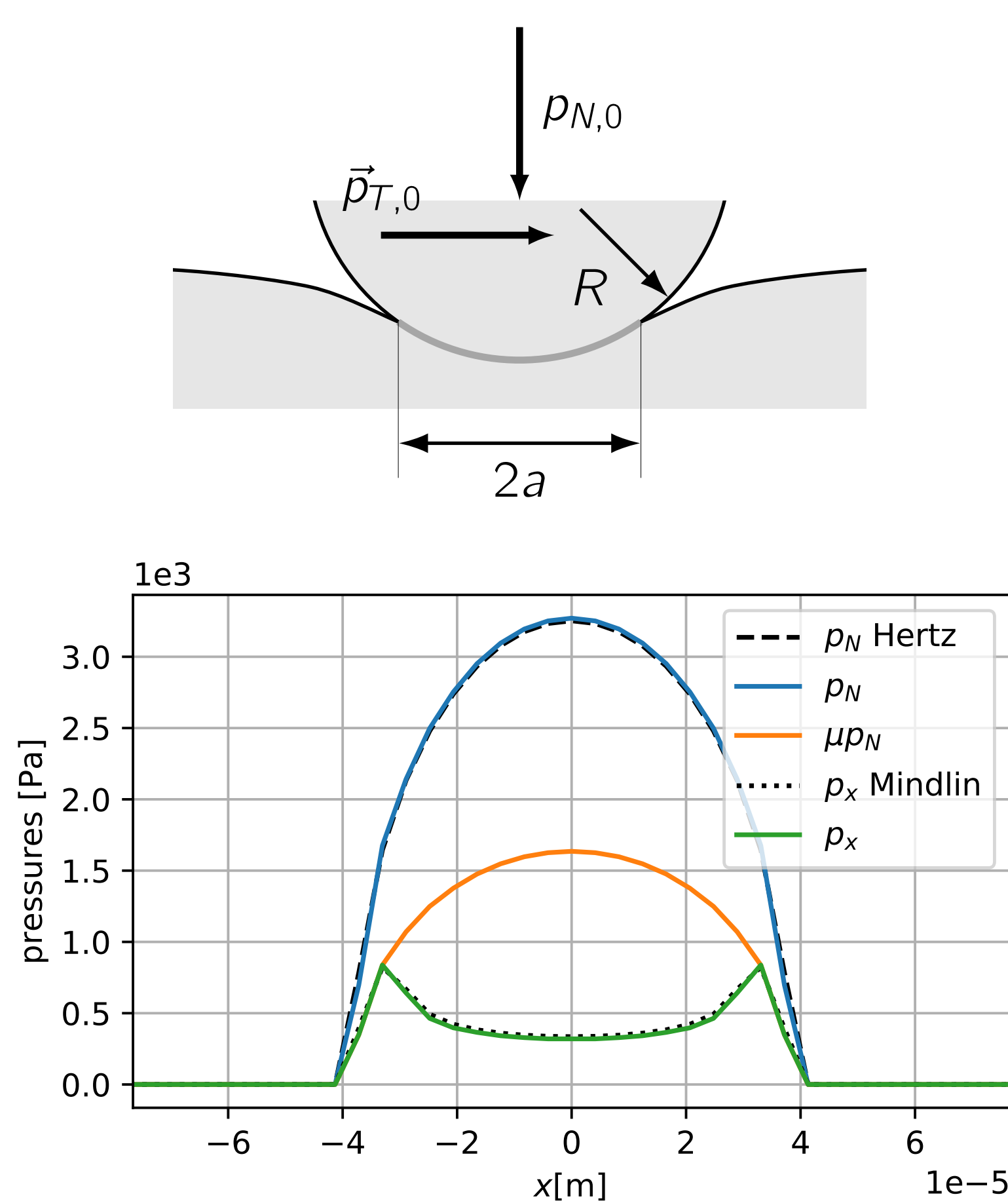
Solving

- Conjugate gradient method [3]
- FFT-based: implies periodic boundary conditions, which are suitable for rough surfaces



Validation

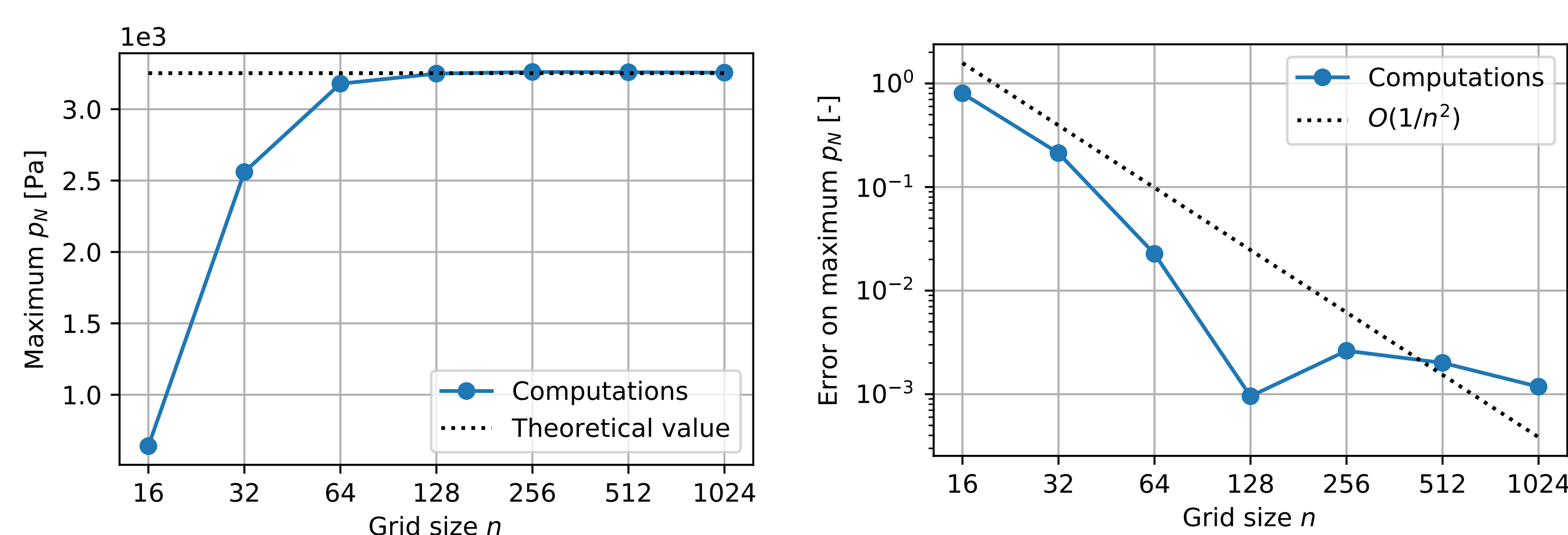
Comparison with Mindlin theory of Hertz contact with Coulomb friction in uncoupled case ($\nu = 0.5$) [2].



Regions:

- contact: $p_N(\mathbf{x}) > 0$
- stick: $\|\vec{p}_T(\mathbf{x})\| = \mu p_N(\mathbf{x})$
- slip: $\|\vec{p}_T(\mathbf{x})\| \leq \mu p_N(\mathbf{x})$

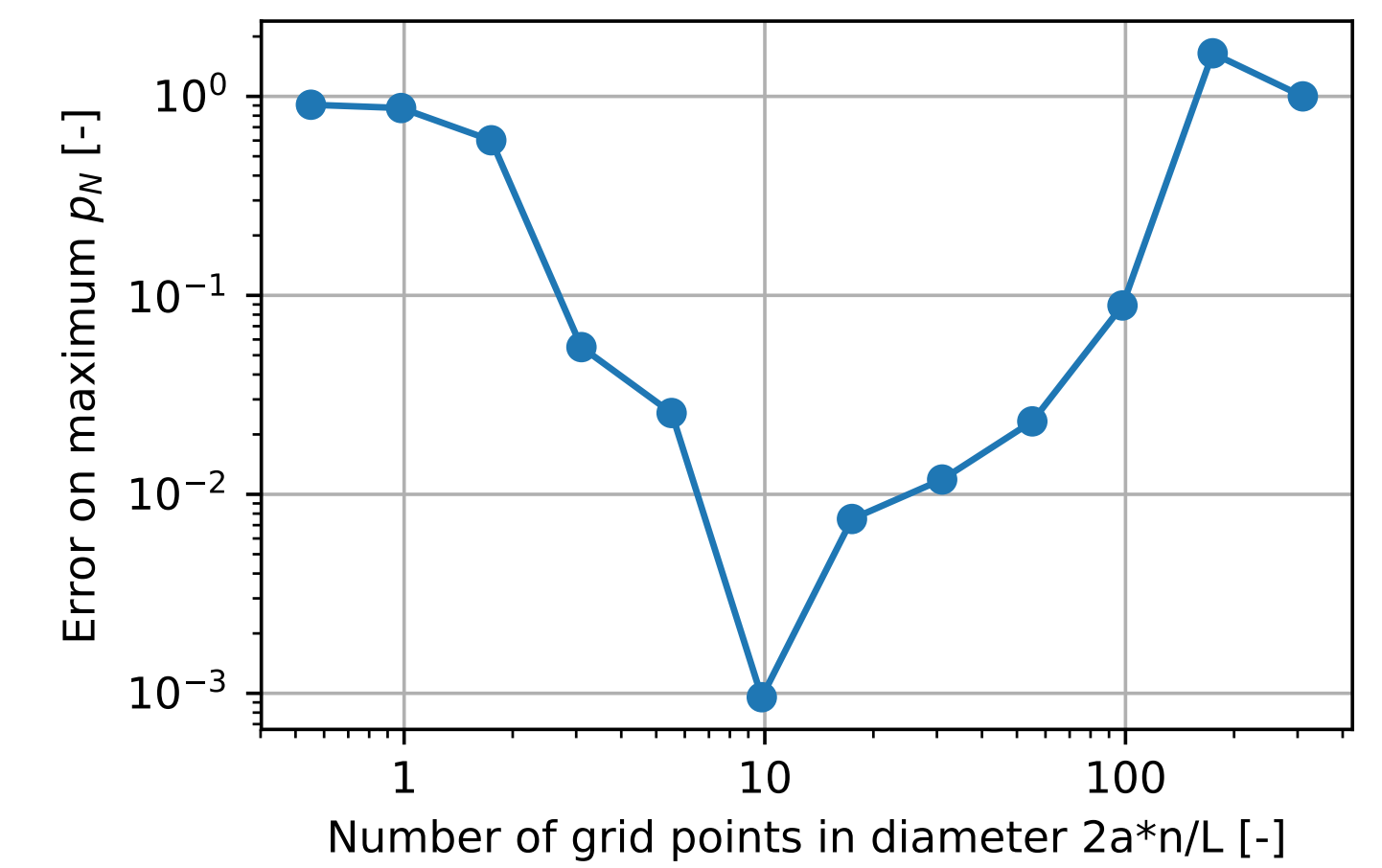
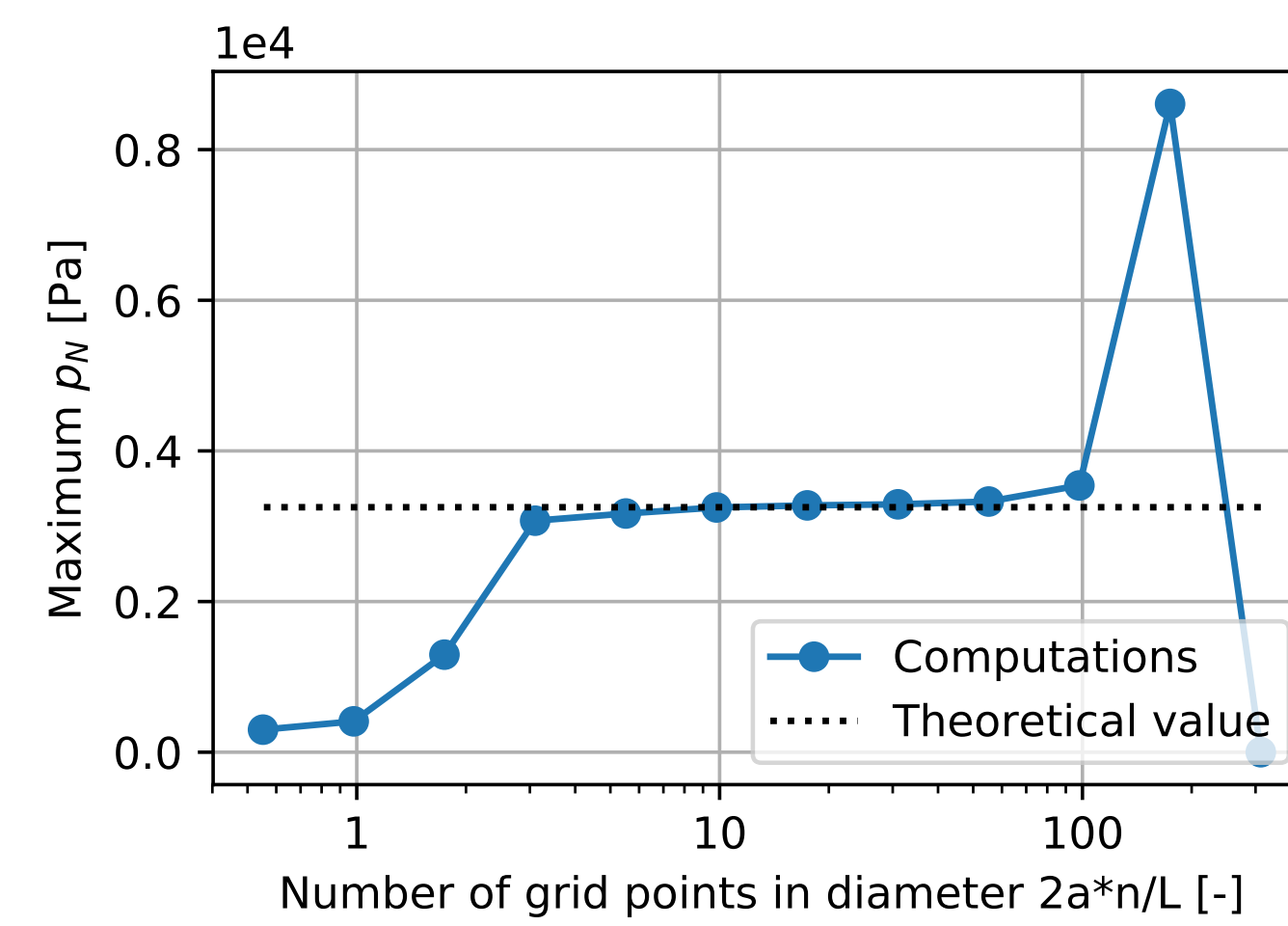
Convergence with mesh refinement (with $L = 1\text{mm}$)



Admissible size of contact zone (with $n = 243$)

The obtained solution converges toward the theoretical solution if:

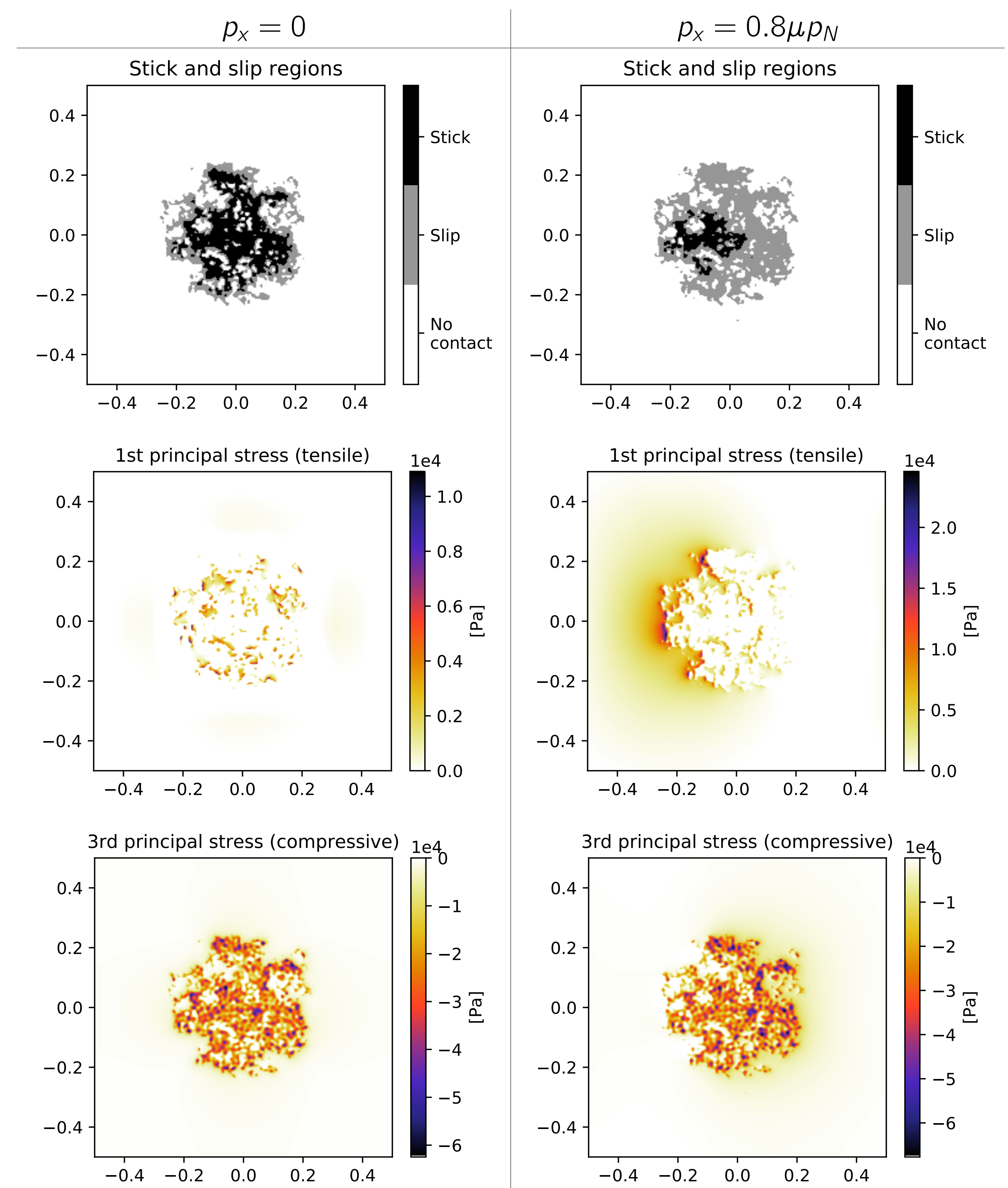
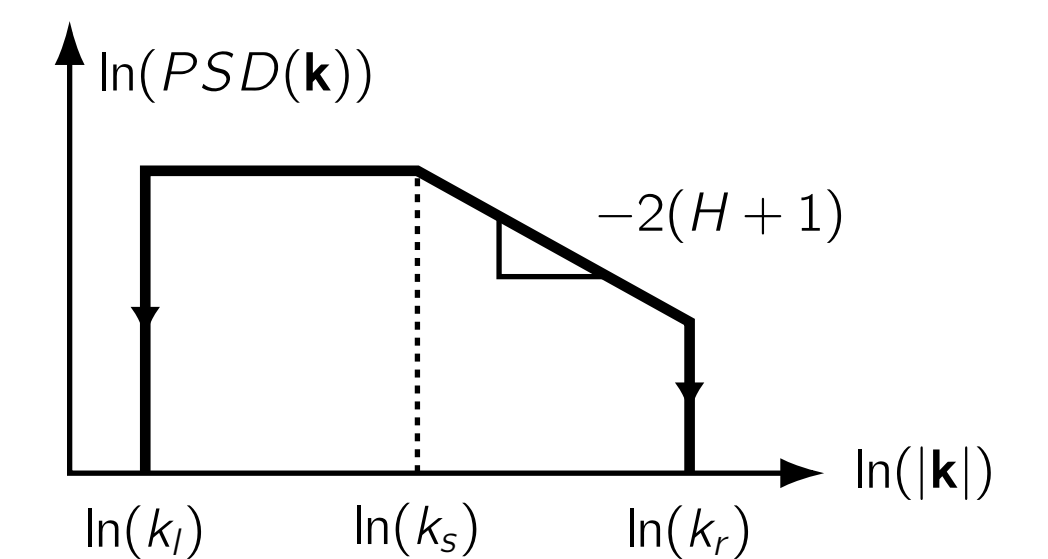
- the contact zone is discretized with enough grid points, $\frac{L}{n} \leq \frac{2a}{10}$,
- the contact zone is small enough to not have boundary effects, $2a \leq \frac{L}{10}$



A more realistic example: rough spherical contact

Parameters:

Material: $E = 1 \cdot 10^6 \text{Pa}$, $\nu = 0.3$, $\mu = 0.5$
 Rough surface [5]: $k_t = k_r = 4$, $k_s = 64$, $H = 0.8$, $\sqrt{\langle |\nabla h|^2 \rangle} = 8 \cdot 10^{-8} \text{m}$
 Hertz contact: $R = 0.01 \text{m}$, $p_{N,0} = 2 \cdot 10^3 \text{Pa}$
 Discretization: $L = 0.001 \text{m}$, $n = 243$



Conclusion

Applications: We have studied the contact asperity patches. Under an increasing normal load, these patches are growing and merging. Adding a tangential load creates stress concentrations only around certain patches and localized slip [4]. The statistics of these contact patches are under investigation.

Limitation: Coulomb friction has no meaning at atomistic scale. We may switch to tangential adhesion formulation and couple it with normal adhesion (consistent with atomic scale interactions).

References

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- [2] K. L. Johnson. *Contact Mechanics*. Cambridge University Press, 1985.
- [3] Valentine Rey, Guillaume Ancaix, and Jean-François Molinari. Normal adhesive contact on rough surfaces: efficient algorithm for FFT-based BEM resolution, Jul 2017.
- [4] R. Sahli, G. Pallares, C. Ducottet, I. E. Ben Ali, S. Al Akhrass, M. Guibert, and J. Scheibert. Evolution of real contact area under shear and the value of static friction of soft materials. *Proceedings of the National Academy of Sciences*, 115(3):471–476, 2018.
- [5] Vladislav A. Yastrebov, Guillaume Ancaix, and Jean-François Molinari. From infinitesimal to full contact between rough surfaces: Evolution of the contact area. *International Journal of Solids and Structures*, 52:83 – 102, 2015.